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Wave equations for a relativistic magnetoplasma

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Abstract. The linearized relativistic Vlasov–Maxwell system for a hot inhomogeneous relativistic magnetized electron plasma is studied through particle orbit theory using the techniques of Fourier transform. An analytical integral expression in the (k, ω)-space is obtained for the current density for waves propagating across an externally applied uniform static magnetic field (k is the wavenumber and ω is the wave frequency). After applying inverse Fourier transform, differential equations for the electric field are obtained from the expression for the current density combined with Maxwell's equations. These fully relativistic equations are correct up to second order in r_c/L, where r_c is the electron gyroradius and L is the gradient length of the plasma inhomogeneity.

1. Introduction

The theory of wave propagation in inhomogeneous hot plasmas is of great practical interest. In nuclear fusion research, for example, the possibilities of heating a plasma by launching an electromagnetic wave are investigated.

So far, most theoretical studies have been concerned with wave propagation in homogeneous magnetoplasmas or in inhomogeneous plasmas in the WKB approximations.

In the study of inhomogeneous hot plasmas Pearson (1966) investigated the propagation of radial electrostatic modes in a plasma column immersed in an axial static magnetic field and developed a set of differential equations for the wave field that is correct up to first order in $r_c|k|$ (r_c is the electron gyroradius, k is the wavenumber). Sivasubramanian and Tang (1972) have derived an integral equation for the wave fields in an unbounded inhomogeneous non-relativistic magnetoplasma. Similarly to Pearson they used particle orbit theory, but unlike Pearson, who used a power-series expansion in the spatial coordinates before the velocity integrations were carried out, they first made a direct Fourier transform and expanded the result into a power series in $r_c|k|$ afterwards. Neither Pearson (1966) nor Sivasubramanian and Tang (1972) considered relativistic effects. Later Imre and Weitzner (1985a, b, c) studied wave propagation in a weakly relativistic plasma near the fundamental electron cyclotron resonance and near the second harmonic. Using geometrical optics and boundary layer theory they derived dispersion relations and absorption coefficients. Recently, Maroli *et al* (1986) and Petrillo *et al* (1987) have done extensive research on the wave-dynamical treatment of the ordinary electron cyclotron mode, taking relativistic effects into account for a specific density and magnetic field profile. In our work we consider an arbitrary density profile and a slowly varying magnetic field and we derive a fully relativistic analytical integral expression for the current density. This result is expanded up to first order in r_c/L afterwards (L is the gradient length of the plasma).

In section 2 we will treat particle orbit theory in the relativistic case and derive an integral expression for the current density. In section 3 we will Fourier transform and apply some elementary asymptotics to derive an expansion of the current density, which then leads to differential equations for the perturbed electric field.

2. Orbit theory

We consider a relativistic, collisionless plasma contained in a fairly strong homogeneous magnetic field $\boldsymbol{B}_0$. It is assumed that its spatial variation can be neglected compared to the spatial variation of the particle density. Being interested in high-frequency perturbations, only the motion of the electrons is of importance. Furthermore it is assumed that the zeroth-order electric field can be neglected. We restrict ourselves to the case of propagation perpendicular to the static magnetic field and furthermore we consider wave propagation parallel to the electron density gradient. The coordinate system is chosen such that $\boldsymbol{B}_0$ lies along the positive z-axis and the inhomogeneity exists only in the x-direction. The relativistic Vlasov equation for the electron momentum distribution function f is

$$\frac{\partial f}{\partial t} + \frac{c\boldsymbol{\rho}}{\gamma(\rho)} \cdot \nabla f - \frac{e}{mc}\left\{\boldsymbol{E} + \frac{c\boldsymbol{\rho}}{\gamma(\rho)} \wedge \boldsymbol{B}\right\} \cdot \nabla_\rho f = 0 \tag{1}$$

where

$$\gamma(\rho) = (\rho^2 + 1)^{1/2} \tag{2}$$

$$\boldsymbol{\rho} = \frac{\boldsymbol{v}}{(c^2 - v^2)^{1/2}} \tag{3}$$

and

$$\nabla_\rho := \left(\frac{\partial}{\partial \rho_x}, \frac{\partial}{\partial \rho_y}, \frac{\partial}{\partial \rho_z}\right). \tag{4}$$

The vector fields $\boldsymbol{E}$ and $\boldsymbol{B}$ denote the electric and magnetic fields, respectively, and m is the electron rest mass. The other symbols have their usual meanings.

The equilibrium solution f_0 is found by solving the unperturbed relativistic Vlasov equation

$$\boldsymbol{\rho} \cdot \nabla f_0 - \frac{1}{c}(\boldsymbol{\rho} \wedge \boldsymbol{\Omega}) \cdot \nabla_\rho f_0 = 0 \tag{5}$$

where the non-relativistic cyclotron frequency is given by

$$\boldsymbol{\Omega} := \frac{e\boldsymbol{B}_0}{m}. \tag{6}$$

Any zeroth-order solution f_0 of (5) is a function of the integrals of motion only (see e.g. Clemmow and Dougherty (1990)). This means

$$f_0(\boldsymbol{x}, \boldsymbol{\rho}) = F_0(\rho, X, \rho_\parallel) \tag{7}$$

where

$$X := x - \frac{c\rho_y}{\Omega}. \tag{8}$$

$\rho_\parallel$ is the component of ρ that is parallel to the magnetic field B_0. In the present case $\rho_\parallel = \rho_z$. The equilibrium distribution function f_0 is chosen to be of the form

$$f_0 = \frac{\mu h(X)}{4\pi K_2(\mu)} \exp\{-\mu\gamma(\rho)\} \tag{9}$$

with

$$\mu = \frac{mc^2}{K_B T} \tag{10}$$

where K_B is Boltzmann's constant, T is the electron temperature and K_2 is the modified Bessel function (Magnus *et al* 1966) of the second kind and of order two. The function h is related to the spatial variation of the density, the other part of f_0 is the relativistic Maxwellian (Synge 1957).

In the present paper we will not elaborate on the precise form of the function h. For that we refer to Kamp *et al* (1992) who have studied several one-dimensional Vlasov–Maxwell equilibria and the conditions under which charge separation and thus zeroth-order electric fields are negligible or even completely absent, as is presently presumed.

The equation for the perturbed electron momentum distribution function f_1 can be written as

$$\frac{\mathrm{d}f_1}{\mathrm{d}t} = \frac{e}{mc}\left\{E_1 + \frac{c\rho}{\gamma(\rho)} \wedge B_1\right\} \cdot \nabla_\rho f_0 \tag{11}$$

where the total derivative on the left-hand side of (11) denotes the time derivative along the unperturbed electron orbits, that are identical with the characteristics of (1). The vector fields E_1 and B_1 denote the perturbed electric and magnetic fields, respectively. We assume that the electromagnetic field vanishes at $t = -\infty$ and consistently $f_1 \equiv 0$ at $t = -\infty$. The solution of (11) is then given by

$$\begin{aligned} f_1\big(x(t), \rho(t)\big) &= f_1\big(x(x_0, \rho_0, t), \rho(x_0, \rho_0, t), t\big) \\ &= \frac{e}{mc}\int_{-\infty}^{t} \mathrm{d}t' \left\{E_1\big(x(t'), t'\big) + \frac{c\rho(t')}{\gamma} \wedge B_1\big(x(t'), t'\big)\right\} \cdot \nabla_\rho f_0\big(x(t'), \rho(t')\big). \end{aligned} \tag{12}$$

The function f_1 depends on $x(t)$ and $\rho(t)$. The current density J in the point (x, t) is found by integrating f_1 multiplied by the relativistic electron velocity and by the Dirac delta-function $\delta\big(x - x(t)\big)$ (in order to evaluate J at the right point) over all possible initial positions x_0 and momenta ρ_0 at $t = -\infty$. Thus we have (see Shafranov (1967))

$$\begin{aligned} J_1(x, t) &= -ec\int \mathrm{d}^3x_0\, \mathrm{d}^3\rho_0\, \delta\big(x - x(t)\big)\frac{\rho(t)}{\gamma} f_1(x_0, \rho_0, t) \\ &= -\frac{e^2}{m}\int \mathrm{d}^3x_0 \mathrm{d}^3\rho_0 \delta\big(x - x(t)\big)\frac{\rho(t)}{\gamma} \\ &\quad \times \int_{-\infty}^{t} \mathrm{d}t' \left\{E_1\big(x(t'), t'\big) + \frac{c\rho(t')}{\gamma} \wedge B_1\big(x(t'), t'\big)\right\} \cdot \nabla_\rho f_0\big(x(t'), \rho(t')\big). \end{aligned} \tag{13}$$

Because the inhomogeneity exists only in the x-direction, the y_0- and z_0-integrations are trivial. To perform the x_0-integration we use the relation

$$x_0 = x(\boldsymbol{x}_0, \boldsymbol{\rho}_0, t) - \frac{c}{\gamma} \int_{-\infty}^{t} \mathrm{d}\tau\, \rho_x(\boldsymbol{x}_0, \boldsymbol{\rho}_0, \tau) . \tag{14}$$

Performing the x_0-integration leads to evaluation of this expression in $x(t) = x$. Thus we have

$$x_0 = x - \frac{c}{\gamma} \int_{-\infty}^{t} \mathrm{d}\tau\, \rho_x(\tau) . \tag{15}$$

This leads to the following relation between $x(t')$ and x:

$$x(t') = x_0 + \frac{c}{\gamma} \int_{-\infty}^{t'} \mathrm{d}\tau\, \rho_x(\tau) = x - \frac{c}{\gamma} \int_{t'}^{t} \mathrm{d}\tau\, \rho_x(\tau) . \tag{16}$$

So we have for $\boldsymbol{J}_1$

$$\begin{aligned} \boldsymbol{J}_1(x,t) &= -\frac{e^2}{m} \int \mathrm{d}^3\rho_0 \frac{\boldsymbol{\rho}(t)}{\gamma} \\ &\quad \times \int_{-\infty}^{t} \mathrm{d}t' \left\{ \boldsymbol{E}_1\big(x(t'),t'\big) + \frac{c\boldsymbol{\rho}(t')}{\gamma} \wedge \boldsymbol{B}_1\big(x(t'),t'\big) \right\} \cdot \nabla_\rho f_0\big(\boldsymbol{x}(t'), \boldsymbol{\rho}(t')\big) \\ &= \frac{e^2\mu}{4\pi m K_2(\mu)} \int \mathrm{d}^3\rho_0 \frac{\boldsymbol{\rho}(t)}{\gamma} \exp(-\mu\gamma) \int_{-\infty}^{t} \mathrm{d}t' \left[\frac{\mu h(X)}{\gamma} \boldsymbol{E}_1\big(x(t'),t'\big) \cdot \boldsymbol{\rho}(t') \right. \\ &\quad \left. + \frac{c}{\Omega} h'(X) \left\{ \boldsymbol{E}_1\big(x(t'),t'\big) + \frac{c\boldsymbol{\rho}(t')}{\gamma} \wedge \boldsymbol{B}_1\big(x(t'),t'\big) \right\} \cdot \mathrm{e}_y \right] \end{aligned} \tag{17}$$

where e_y is the unit vector in the y-direction.

3. Fourier transform of the current density

In this section the Fourier transform of the current density is performed. Thus, an integral expression is obtained, which is expanded up to second order in r_c/L. Then the inverse Fourier transform of the current density combined with Maxwell's equations gives us a set of differential equations for the perturbed electric field. The one-dimensional Fourier transform both in space and in time variables of a function ϕ is defined by

$$\phi(k,\omega) := \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}t \exp(\mathrm{i}\omega t) \int_{-\infty}^{\infty} \mathrm{d}x \exp(-\mathrm{i}kx) \phi(x,t) . \tag{18}$$

To avoid an abundance of notation the Fourier transform and the function itself are both denoted by ϕ. The argument shows whether the function or its Fourier transform is meant. The Fourier transform of $\boldsymbol{J}$ is given by

$$\begin{aligned} \boldsymbol{J}(k,\omega) &= \frac{e^2\mu}{8\pi^2 m K_2(\mu)} \int_{-\infty}^{\infty} \mathrm{d}t \exp(\mathrm{i}\omega t) \int_{-\infty}^{\infty} \mathrm{d}x \exp(-\mathrm{i}kx) \\ &\quad \times \int \mathrm{d}^3\rho_0 \frac{\boldsymbol{\rho}(t)}{\gamma} \exp(-\mu\gamma) \int_{-\infty}^{t} \mathrm{d}t' \left[\frac{\mu h(X)}{\gamma} \boldsymbol{E}_1\big(x(t'),t'\big) \cdot \boldsymbol{\rho}(t') \right. \\ &\quad \left. + \frac{c}{\Omega} h'(X) \left\{ \boldsymbol{E}_1\big(x(t'),t'\big) + \frac{c\boldsymbol{\rho}(t')}{\gamma} \wedge \boldsymbol{B}_1\big(x(t'),t'\big) \right\} \cdot \mathrm{e}_y \right] . \end{aligned} \tag{19}$$

Because ρ and ρ_0 are independent of x and the function h is time independent, the order of some of the integrations can be interchanged. This yields

$$J_1(k,\omega) = \frac{e^2\mu}{8\pi^2 m K_2(\mu)} \int d^3\rho_0 \frac{1}{\gamma} \exp(-\mu\gamma) \int_{-\infty}^{\infty} dt \exp(i\omega t)\rho(t) \times \int_{-\infty}^{t} dt' \int_{-\infty}^{\infty} dx \exp(-ikx) \left[\frac{\mu h(X)}{\gamma} E_1(x(t'),t') \cdot \rho(t') + \frac{c}{\Omega} h'(X) \left\{ E_1(x(t'),t') + \frac{c\rho(t')}{\gamma} \wedge B_1(x(t'),t') \right\} \cdot e_y \right]. \qquad (20)$$

First we calculate the integral

$$\int_{-\infty}^{\infty} dx \exp(-ikx) h(X) E_1\{x(t'),t'\} = \int_{-\infty}^{\infty} dx \exp(-ikx) h^{(1)}(x) E_1^{(1)}(x,t') \qquad (21)$$

with

$$h^{(1)}(x) := h(X) \qquad E_1^{(1)}(x,t') := E_1\{x(t'),t'\}. \qquad (22)$$

According to the convolution theorem for the Fourier transform (Sneddon 1951) we find for the right-hand side of (21)

$$\int_{-\infty}^{\infty} dx \exp(-ikx) h^{(1)}(x) E_1^{(1)}(x,t') = \int_{-\infty}^{\infty} dk' h^{(1)}(k-k') E_1^{(1)}(k',t') \qquad (23)$$

with

$$\begin{aligned} h^{(1)}(k-k') &= \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} dx \exp\{-i(k-k')x\} h^{(1)}(x) \\ &= \frac{1}{(2\pi)^{1/2}} \exp\left\{-i\frac{c}{\Omega}(k-k')\rho_y(t)\right\} \int_{-\infty}^{\infty} dx \exp\{-i(k-k')x\} h(x) \\ &= h(k-k') \exp\left\{-i\frac{c}{\Omega}(k-k')\rho_y(t)\right\} \end{aligned} \qquad (24)$$

and

$$\begin{aligned} E_1^{(1)}(k',t') &= \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} dx \exp(-ik'x) E_1^{(1)}(x,t') \\ &= \frac{1}{(2\pi)^{1/2}} \exp\left\{-i\frac{k'c}{\gamma} \int_{t'}^{t} d\tau \rho_x(\tau)\right\} \int_{-\infty}^{\infty} d\{x(t')\} \exp\{-ik'x(t')\} E_1(x(t'),t') \\ &= E_1(k',t') \exp\left\{-i\frac{k'c}{\gamma} \int_{t'}^{t} d\tau \rho_x(\tau)\right\}. \end{aligned} \qquad (25)$$

In this way the following expression is obtained for J_1:

$$J_{1i}(k,\omega) = \frac{e^2\mu}{8\pi^2 m K_2(\mu)} \int_{-\infty}^{\infty} dk' h(k-k') \int_{-\infty}^{\infty} dt \exp(-i\omega t) \int_{-\infty}^{t} dt' \times \int d^3\rho_0 \frac{\rho_i(t)}{\gamma} \exp\left\{-\mu\gamma - i\frac{c}{\Omega}(k-k')\rho_y(t) - i\frac{ck'}{\gamma}\right\} \int_{t'}^{t} d\tau\, \rho_x(\tau) \times \left[\frac{\mu}{\gamma}\rho_j(t') E_{1j}(k',t') + i\frac{c}{\Omega}(k-k') \left\{ E_{1y}(k',t') - \frac{c}{\gamma} B_{1z}(k',t') \right\} \right]. \qquad (26)$$

Here we have used tensor notation ($i, j = x, y, z$) and the Einstein convention (repeated indices denote summation). In appendix A the various components of the current density are calculated. The following expression is obtained:

$$J_{1i}(\kappa,\omega) = \frac{\mathrm{i}e^2}{(2\pi)^{1/2}m\omega L}\int_{-\infty}^{\infty}\mathrm{d}\kappa'\bigg\{R_{ij}(\kappa,\kappa')E_{1j}(\kappa',\omega)h(\kappa-\kappa') + \delta_{iy}\frac{r_c^2}{L^2}(\kappa-\kappa')^2 n_0(\kappa-\kappa')E_{1y}(\kappa',\omega)\bigg\} \tag{27}$$

with

$$R_{ij} = \begin{pmatrix} \sum_{n=-\infty}^{\infty} n^2\nu^{-1}f_n^{(\frac{3}{2})} & \sum_{n=-\infty}^{\infty} -\frac{\mathrm{i}nL^2}{r_c^2\kappa}\frac{\partial f_n^{(\frac{3}{2})}}{\partial\kappa'} & 0 \\ \sum_{n=-\infty}^{\infty}\frac{\mathrm{i}nL^2}{r_c^2\kappa'}\frac{\partial f_n^{(\frac{3}{2})}}{\partial\kappa} & \sum_{n=-\infty}^{\infty}\frac{L^2}{r_c^2}\frac{\partial^2 f_n^{(\frac{3}{2})}}{\partial\kappa\partial\kappa'} & 0 \\ 0 & 0 & \sum_{n=-\infty}^{\infty} f_n^{(\frac{5}{2})} \end{pmatrix} \tag{28}$$

in which the function $f_n^{(r)}$ is given by

$$f_n^{(r)}(\kappa,\kappa') = \left(\frac{\pi\mu}{2}\right)^{1/2}\frac{\exp(-\mu)}{K_2(\mu)}\frac{(2|n|)!}{(|n|)!}\left(\frac{\nu}{2}\right)^{|n|}\sum_{m=0}^{\infty}\frac{(-1)^m(\kappa^2-\kappa'^2)^m}{\Gamma(2|n|+m+1)}\left(\frac{r_c^2}{2L^2}\right)^m \times \mathrm{C}_m^{(|n|+\frac{1}{2})}\left(\frac{\kappa^2+\kappa'^2}{\kappa^2-\kappa'^2}\right)\mathcal{F}_{m+|n|+r}\left(\mu,\frac{n\Omega}{\omega}\right). \tag{29}$$

Here $\kappa = kL$, $\kappa' = k'L$, $\nu = r_c^2\kappa\kappa'/L^2$ and δ_{iy} is the Kronecker delta. The function $\mathrm{C}_m^{(|n|+\frac{1}{2})}$ is the Gegenbauer polynomial (Magnus *et al* 1966) and the function $\mathcal{F}_{m+|n|+r}(\mu, n\Omega/\omega)$ is the relativistic dispersion function (Bornatici and Ruffina 1988) defined by

$$\mathcal{F}_q(\mu,\alpha) = \left(\frac{\mu}{2}\right)^{q-1}\frac{1}{\Gamma(q)}\int_1^{\infty}\mathrm{d}\gamma\,\frac{(\gamma^2-1)^{q-1}}{\gamma-\alpha}\exp\{-\mu(\gamma-1)\} \qquad \alpha\in C \quad \mu>0. \tag{30}$$

In appendix B some properties of the function $\mathcal{F}_q$ are presented.

In the limit $L \to \infty$ the elements of the tensor R_{ij} still exist and coincide with the cold-plasma limit. The non-relativistic limit of $\mu\mathcal{F}_q(\mu, n\Omega/\omega)$ (i.e. $\mu \to \infty$) is equal to $(1 - n\Omega/\omega)^{-1}$ and the non-relativistic limit of $(\pi/2\mu)^{1/2}\exp(-\mu)/K_2(\mu)$ is unity. Therefore the non-relativistic limit of $f_n^{(r)}$ reads

$$f_n^{(r)}(\kappa,\kappa') = \frac{(2|n|)!}{(|n|)!}\frac{1}{1-n\Omega/\omega}\left(\frac{\nu}{2}\right)^{|n|}\sum_{m=0}^{\infty}\frac{-r_c^2(\kappa^2-\kappa'^2)^m}{\Gamma(2|n|+m+1)}\mathrm{C}_m^{(|n|+\frac{1}{2})}\left(\frac{\kappa^2+\kappa'^2}{\kappa^2-\kappa'^2}\right). \tag{31}$$

Using the following summation formula for the Gegenbauer polynomial (Magnus *et al* 1966):

$$\sum_{m=0}^{\infty}\frac{\mathrm{C}_m^{(\lambda)}\{\cos(\theta)\}}{(2\lambda)_m}z^m = \Gamma\left(\lambda+\frac{1}{2}\right)\exp\{z\cos(\theta)\}\left\{\frac{z}{2}\sin(\theta)\right\}^{\frac{1}{2}-\lambda}J_{\lambda-\frac{1}{2}}\{z\sin(\theta)\} \tag{32}$$

expression (31) results in

$$f_n^{(r)}(\kappa,\kappa') = \frac{I_n(\nu)}{(1-n\Omega/\omega)}\exp\left\{-\frac{r_c^2(\kappa^2+\kappa'^2)}{2L^2}\right\} \tag{33}$$

where I_n is the modified Bessel function (Magnus *et al* 1966) of the first kind and of order n. The results described by Sivasubramanian and Tang (1972) are thus recovered.

4. Small gyroradius approximation

In this section the elements of the tensor **R** as defined in section 3 are expanded up to second order in r_c/L. Using this expansion for the tensor **R**, the inverse Fourier transform of the current density can be carried out and subsequent substitution of the result in the Maxwell equations yields the following set of fully relativistic differential equations for the electric field. Two coupled second-order differential equations for the X-mode and one second-order differential equation for the O-mode, namely

$$\left(\mathsf{I} + \boldsymbol{\Sigma}_1 + \frac{r_c^2}{L^2}\boldsymbol{\Sigma}_2\right) \cdot \boldsymbol{E}_1 = 0 \tag{34}$$

where the non-zero elements of the tensor operators $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ are given by

$$\Sigma_{1,xx} = -\left(\frac{\pi\mu}{2}\right)^{1/2} \exp(-\mu)\frac{\omega_p^2}{2K_2(\mu)\omega^2}\left\{\mathcal{F}_{\frac{5}{2}}\left(\mu, \frac{\Omega}{\omega}\right) + \mathcal{F}_{\frac{5}{2}}\left(\mu, -\frac{\Omega}{\omega}\right)\right\} \tag{35}$$

$$\Sigma_{1,xy} = \mathrm{i}\left(\frac{\pi\mu}{2}\right)^{1/2} \exp(-\mu)\frac{\omega_p^2}{2K_2(\mu)\omega^2}\left\{\mathcal{F}_{\frac{5}{2}}\left(\mu, \frac{\Omega}{\omega}\right) - \mathcal{F}_{\frac{5}{2}}\left(\mu, -\frac{\Omega}{\omega}\right)\right\} \tag{36}$$

$$\Sigma_{1,yx} = -\mathrm{i}\left(\frac{\pi\mu}{2}\right)^{1/2} \exp(-\mu)\frac{\omega_p^2}{2K_2(\mu)\omega^2}\left\{\mathcal{F}_{\frac{5}{2}}\left(\mu, \frac{\Omega}{\omega}\right) - \mathcal{F}_{\frac{5}{2}}\left(\mu, -\frac{\Omega}{\omega}\right)\right\} \tag{37}$$

$$\Sigma_{1,yy} = \frac{c^2}{\omega^2 L^2}\frac{\partial^2}{\partial\xi^2} - \left(\frac{\pi\mu}{2}\right)^{1/2} \exp(-\mu)\frac{\omega_p^2}{2K_2(\mu)\omega^2}\left\{\mathcal{F}_{\frac{5}{2}}\left(\mu, \frac{\Omega}{\omega}\right) + \mathcal{F}_{\frac{5}{2}}\left(\mu, -\frac{\Omega}{\omega}\right)\right\} \tag{38}$$

$$\Sigma_{1,zz} = \frac{c^2}{\omega^2 L^2}\frac{\partial^2}{\partial\xi^2} - \left(\frac{\pi\mu}{2}\right)^{1/2} \exp(-\mu)\frac{\omega_p^2}{2K_2(\mu)\omega^2}\mathcal{F}_{\frac{5}{2}}(\mu, 0) \tag{39}$$

$$\begin{aligned}\Sigma_{2,xx} = \left(\frac{\pi\mu}{2}\right)^{1/2} \frac{\exp(-\mu)}{2K_2(\mu)\omega^2}\Bigg[&\frac{1}{4}\left\{\mathcal{F}_{\frac{5}{2}}\left(\mu, \frac{\Omega}{\omega}\right) + \mathcal{F}_{\frac{5}{2}}\left(\mu, -\frac{\Omega}{\omega}\right)\right\}\frac{K_3(\mu)}{K_2(\mu)}(\omega_p^2)'' \\ &- \frac{1}{2}\left\{\mathcal{F}_{\frac{7}{2}}\left(\mu, \frac{\Omega}{\omega}\right) + \mathcal{F}_{\frac{7}{2}}\left(\mu, -\frac{\Omega}{\omega}\right)\right\}\left\{\frac{1}{2}(\omega_p^2)'' + \frac{\partial}{\partial\xi}\left(\omega_p^2\frac{\partial}{\partial\xi}\right)\right\} \\ &+ \frac{1}{2}\left\{\mathcal{F}_{\frac{7}{2}}\left(\mu, \frac{2\Omega}{\omega}\right) + \mathcal{F}_{\frac{7}{2}}\left(\mu, -\frac{2\Omega}{\omega}\right)\right\}\frac{\partial}{\partial\xi}\left(\omega_p^2\frac{\partial}{\partial\xi}\right)\Bigg]\end{aligned} \tag{40}$$

$$\begin{aligned}\Sigma_{2,xy} = \mathrm{i}\left(\frac{\pi\mu}{2}\right)^{1/2} \frac{\exp(-\mu)}{2K_2(\mu)\omega^2}\Bigg[&-\frac{1}{4}\left\{\mathcal{F}_{\frac{5}{2}}\left(\mu, \frac{\Omega}{\omega}\right) - \mathcal{F}_{\frac{5}{2}}\left(\mu, -\frac{\Omega}{\omega}\right)\right\}\frac{K_3(\mu)}{K_2(\mu)}(\omega_p^2)'' \\ &+ \frac{1}{4}\left\{\mathcal{F}_{\frac{7}{2}}\left(\mu, \frac{\Omega}{\omega}\right) - \mathcal{F}_{\frac{7}{2}}\left(\mu, -\frac{\Omega}{\omega}\right)\right\}\left\{(\omega_p^2)'' + 2(\omega_p^2)'\frac{\partial}{\partial\xi} + 4\omega_p^2\frac{\partial^2}{\partial\xi^2}\right\} \\ &- \frac{1}{2}\left\{\mathcal{F}_{\frac{7}{2}}\left(\mu, \frac{2\Omega}{\omega}\right) - \mathcal{F}_{\frac{7}{2}}\left(\mu, -\frac{2\Omega}{\omega}\right)\right\}\frac{\partial}{\partial\xi}\left(\omega_p^2\frac{\partial}{\partial\xi}\right)\Bigg]\end{aligned} \tag{41}$$

$$\begin{aligned}\Sigma_{2,yx} = -\mathrm{i}\left(\frac{\pi\mu}{2}\right)^{1/2} \frac{\exp(-\mu)}{2K_2(\mu)\omega^2}\Bigg[&-\frac{1}{4}\left\{\mathcal{F}_{\frac{5}{2}}\left(\mu, \frac{\Omega}{\omega}\right) - \mathcal{F}_{\frac{5}{2}}\left(\mu, -\frac{\Omega}{\omega}\right)\right\}\frac{K_3(\mu)}{K_2(\mu)}(\omega_p^2)'' \\ &+ \frac{1}{4}\left\{\mathcal{F}_{\frac{7}{2}}\left(\mu, \frac{\Omega}{\omega}\right) - \mathcal{F}_{\frac{7}{2}}\left(\mu, -\frac{\Omega}{\omega}\right)\right\}\left\{(3\omega_p^2)'' + 6(\omega_p^2)'\frac{\partial}{\partial\xi} + 4\omega_p^2\frac{\partial^2}{\partial\xi^2}\right\}\end{aligned}$$

$$-\frac{1}{2}\left\{\mathcal{F}_{\frac{7}{2}}\left(\mu,\frac{2\Omega}{\omega}\right)-\mathcal{F}_{\frac{7}{2}}\left(\mu,-\frac{2\Omega}{\omega}\right)\right\}\frac{\partial}{\partial\xi}\left(\omega_{\rm p}^2\frac{\partial}{\partial\xi}\right)\Bigg] \tag{42}$$

$$\Sigma_{2,yy}=-\left(\frac{\pi\mu}{2}\right)^{1/2}\frac{\exp(-\mu)}{2K_2(\mu)\omega^2}\Bigg[-\frac{1}{4}\left\{\mathcal{F}_{\frac{5}{2}}\left(\mu,\frac{\Omega}{\omega}\right)-\mathcal{F}_{\frac{5}{2}}\left(\mu,-\frac{\Omega}{\omega}\right)\right\}\frac{K_3(\mu)}{K_2(\mu)}(\omega_{\rm p}^2)''$$
$$+\frac{3}{4}\left\{\mathcal{F}_{\frac{7}{2}}\left(\mu,\frac{\Omega}{\omega}\right)-\mathcal{F}_{\frac{7}{2}}\left(\mu,-\frac{\Omega}{\omega}\right)\right\}\left\{(\omega_{\rm p}^2)''+2(\omega_{\rm p}^2)'\frac{\partial}{\partial\xi}+2\omega_{\rm p}^2\frac{\partial^2}{\partial\xi^2}\right\}$$
$$-\left\{\mathcal{F}_{\frac{7}{2}}\left(\mu,\frac{2\Omega}{\omega}\right)+\mathcal{F}_{\frac{7}{2}}\left(\mu,-\frac{2\Omega}{\omega}\right)\right\}\left\{\frac{\partial}{\partial\xi}\left(\omega_{\rm p}^2\frac{\partial}{\partial\xi}\right)-(\omega_{\rm p}^2)''\right\}\Bigg] \tag{43}$$

$$\Sigma_{2,zz}=-\left(\frac{\pi\mu}{2}\right)^{1/2}\frac{\exp(-\mu)}{2K_2(\mu)\omega^2}\Bigg[-\frac{1}{2}\mathcal{F}_{\frac{5}{2}}(\mu,0)\frac{K_3(\mu)}{K_2(\mu)}(\omega_{\rm p}^2)''$$
$$+\frac{1}{2}\mathcal{F}_{\frac{7}{2}}(\mu,0)\left\{(\omega_{\rm p}^2)'\frac{\partial}{\partial\xi}+2\frac{\partial}{\partial\xi}\left(\omega_{\rm p}^2\frac{\partial}{\partial\xi}\right)\right\}$$
$$-\frac{1}{2}\left\{\mathcal{F}_{\frac{7}{2}}\left(\mu,\frac{\Omega}{\omega}\right)+\mathcal{F}_{\frac{7}{2}}\left(\mu,-\frac{\Omega}{\omega}\right)\right\}\frac{\partial}{\partial\xi}\left(\omega_{\rm p}^2\frac{\partial}{\partial\xi}\right)\Bigg]. \tag{44}$$

Here ξ denotes the dimensionless coordinate x/L.

5. Oblique incidence, perpendicular to the magnetic field

In this section we consider a similar geometry as earlier in this paper, with this difference that the wave vector has a component k_y in the y-direction. We have

$$\boldsymbol{E}_1,\boldsymbol{B}_1\propto\exp(\mathrm{i}k_y y). \tag{45}$$

For the first-order current density we now obtain the following expression:

$$\boldsymbol{J}_1(x,y,t)=\frac{e^2\mu}{4\pi K_2(\mu)}\int\mathrm{d}^3\rho_0\frac{\boldsymbol{\rho}(t)}{\gamma}\exp(-\mu\gamma)$$
$$\times\int_{-\infty}^{t}\mathrm{d}t'\Bigg[\frac{\mu h(X)}{\gamma}\boldsymbol{E}_1\big(x(t'),t'\big)\cdot\boldsymbol{\rho}(t)\exp\{\mathrm{i}k_y y(t')\}+\frac{c}{\Omega}h'(X)\exp\{\mathrm{i}k_y y(t')\}$$
$$\times\left\{\boldsymbol{E}_1\big(x(t'),t'\big)+\frac{c\boldsymbol{\rho}(t')}{\gamma}\wedge\boldsymbol{B}_1\big(x(t'),t'\big)\right\}\cdot\boldsymbol{e}_y\Bigg] \tag{46}$$

where the relation between x, y and $x(t')$, $y(t')$ is given by

$$x(t')=x-\frac{c}{\gamma}\int_{t'}^{t}\mathrm{d}\tau\rho_x(\tau)\qquad y(t')=y-\frac{c}{\gamma}\int_{t'}^{t}\mathrm{d}\tau\rho_y(\tau). \tag{47}$$

We Fourier transform with respect to x and t and we introduce the dimensionless wavenumbers κ_x,κ_y given by $\kappa_i=k_iL$, $i=x,y$. In this way we obtain

$$J_{1i}(\boldsymbol{\kappa},\omega)=\frac{e^2\mu}{8\pi^2 mLK_2(\mu)}\exp\left(\mathrm{i}\kappa_y\frac{y}{L}\right)\int_{-\infty}^{\infty}\mathrm{d}\kappa_x'h(\kappa_x-\kappa_x')\int_{-\infty}^{\infty}\mathrm{d}t\exp(\mathrm{i}\omega t)$$
$$\times\int_{-\infty}^{t}\mathrm{d}t'\int\mathrm{d}^3\rho_0\frac{\rho_i(t)}{\gamma}\exp\Bigg\{-\mu\gamma-\mathrm{i}\frac{c}{\Omega L}(\kappa_x-\kappa_x')\rho_y(t)$$
$$-\mathrm{i}\frac{c\kappa_x'}{\gamma}\int_{t'}^{t}\mathrm{d}\tau\rho_x(\tau)-\mathrm{i}\frac{c\kappa_y}{\gamma L}\int_{t'}^{t}\mathrm{d}\tau\rho_y(\tau)\Bigg\}$$
$$\times\left[\frac{\mu}{\gamma}\rho_j(t')E_{1j}(\kappa_x',t')+\mathrm{i}\frac{c}{\Omega L}(\kappa_x-\kappa_x')\left\{E_{1y}(\kappa_x',t')-\frac{c}{\gamma}\rho_x(t')B_{1z}(\kappa_x',t')\right\}\right]. \tag{48}$$

This expression is calculated in appendix A. In the case $k_y = 0$ the expression for the current density reduces to that of section 3 (i.e. (26)).

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Appendix A. Calculation of the current density

In section 3 the following expression was obtained for the current density (i.e. (48)):

$$\begin{aligned} J_{1i}(\kappa,\omega) = {} & \frac{e^2\mu}{8\pi^2 mLK_2(\mu)}\exp\left(\mathrm{i}\kappa_y\frac{y}{L}\right)\int_{-\infty}^{\infty}\mathrm{d}\kappa_x' h(\kappa_x-\kappa_x')\int_{-\infty}^{\infty}\mathrm{d}t\exp(\mathrm{i}\omega t) \\ & \times\int_{-\infty}^{t}\mathrm{d}t'\int\mathrm{d}^3\rho_0\frac{\rho_i(t)}{\gamma}\exp\left\{-\mu\gamma-\mathrm{i}\frac{c}{\Omega L}(\kappa_x-\kappa_x')\rho_y(t)\right. \\ & \left.-\mathrm{i}\frac{c\kappa_x'}{\gamma}\int_{t'}^{t}\mathrm{d}\tau\rho_x(\tau)-\mathrm{i}\frac{c\kappa_y}{\gamma L}\int_{t'}^{t}\mathrm{d}\tau\rho_y(\tau)\right\} \\ & \times\left[\frac{\mu}{\gamma}\rho_j(t')E_{1j}(\kappa_x',t')+\mathrm{i}\frac{c}{\Omega L}(\kappa_x-\kappa_x')\left\{E_{1y}(\kappa_x',t')-\frac{c}{\gamma}\rho_x(t')B_{1z}(\kappa_x',t')\right\}\right]. \end{aligned} \tag{A1}$$

(The parameter κ_y' is a dummy variable. Further on in the present appendix we choose $\kappa_y' = \kappa_y$.) By introducing the notations

$$\begin{aligned} \kappa_\perp &= \left(\kappa_x^2+\kappa_y^2\right)^{1/2} \qquad & \kappa_\perp' &= \left\{\left(\kappa_x'\right)^2+\left(\kappa_y'\right)^2\right\}^{1/2} \\ \phi_\kappa &= \arctan\left(\frac{\kappa_y}{\kappa_x}\right) \qquad & \phi_{\kappa'} &= \arctan\left(\frac{\kappa_y'}{\kappa_x'}\right) \end{aligned} \tag{A2}$$

we define an integral I_1 as follows:

$$\begin{aligned} I_1 &= \int_0^{2\pi}\mathrm{d}\phi\exp\left[-\mathrm{i}\frac{c\rho_\perp}{\Omega L}\left\{\kappa_\perp\sin\left(\phi-\phi_\kappa+\frac{\Omega}{\gamma}t\right)-\kappa_\perp'\sin\left(\phi-\phi_{\kappa'}+\frac{\Omega}{\gamma}t'\right)\right\}\right] \\ &= \int_0^{2\pi}\mathrm{d}\phi\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}J_m\left(\frac{c\kappa_\perp\rho_\perp}{\Omega L}\right)J_n\left(\frac{c\kappa_\perp'\rho_\perp}{\Omega L}\right) \\ &\quad\times\exp\left[-\mathrm{i}\left\{m\left(\phi-\phi_\kappa+\frac{\Omega}{\gamma}t\right)-n\left(\phi-\phi_{\kappa'}+\frac{\Omega}{\gamma}t'\right)\right\}\right] \\ &= 2\pi\sum_{n=-\infty}^{\infty}J_n\left(\frac{c\kappa_\perp\rho_\perp}{\Omega L}\right)J_n\left(\frac{c\kappa_\perp'\rho_\perp}{\Omega L}\right)\exp\left[\mathrm{i}n\left\{\phi_\kappa-\phi_{\kappa'}+\frac{\Omega}{\gamma}(t-t')\right\}\right] \end{aligned} \tag{A3}$$

where J_n is the Bessel function (Magnus *et al* 1966) of the first kind and of order n. Consider now an integral I_2 given by

$$I_2(\omega,n\Omega) = \int_{-\infty}^{\infty}\mathrm{d}t\exp(\mathrm{i}\omega t)\int_{-\infty}^{t}\mathrm{d}t' E_{1i}(\kappa_x',t')\exp\left\{\mathrm{i}\frac{n\Omega}{\gamma}(t-t')\right\}. \tag{A4}$$

Assuming that $\mathrm{Im}\,\omega > 0$ (causality condition) we have

$$
\begin{aligned}
I_2(\omega, n\Omega) &= \int_{-\infty}^{\infty} \mathrm{d}t' E_{1i}(\kappa_x', t') \exp\left(\mathrm{i}\frac{n\Omega}{\gamma}t'\right) \int_{t'}^{\infty} \mathrm{d}t \exp\left\{-\mathrm{i}\left(\frac{n\Omega}{\gamma} - \omega\right)t\right\} \\
&= \int_{-\infty}^{\infty} \mathrm{d}t' E_{1i}(\kappa_x', t') \exp\left(\mathrm{i}\frac{n\Omega}{\gamma}t'\right) \times \frac{-\mathrm{i}}{n\Omega/\gamma - \omega} \exp\left\{-\mathrm{i}\left(\frac{n\Omega}{\gamma} - \omega\right)t'\right\} \\
&= \frac{\mathrm{i}}{\omega}(2\pi)^{1/2} \frac{E_{1i}(\kappa_x', \omega)}{1 - n\Omega/\gamma\omega}\,.
\end{aligned}
\tag{A5}
$$

We also calculate I_3, defined by

$$
I_3 = 2\int_0^{\pi/2} \mathrm{d}\psi \cos(\psi) J_n\{\alpha\rho\cos(\psi)\} J_n\{\beta\rho\cos(\psi)\}\,. \tag{A6}
$$

The integral I_3 is equal to

$$
\begin{aligned}
&\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \left(\frac{\alpha}{2}\right)^{|n|+2k} \left(\frac{\beta}{2}\right)^{|n|+2l} \frac{(-1)^{k+l}\rho^{2|n|+2k+2l}}{k!\,l!\,(k+|n|)!\,(l+|n|)!} \frac{\Gamma(|n|+k+l+1)\Gamma(1/2)}{\Gamma(|n|+k+l+3/2)} \\
&\quad = (\alpha\beta)^{|n|} 2^{-2|n|} \rho^{2|n|} (2\pi)^{1/2} \sum_{m=0}^{\infty} \frac{(-1)^m \rho^{2m}}{\Gamma(m+|n|+\frac{3}{2})} \frac{2^{-2m}}{\Gamma(m+|n|+1)} \\
&\quad\quad \times \sum_{k=0}^{m} \binom{|n|+m}{k} \binom{|n|+m}{|n|-k} \alpha^{2k}\beta^{2(m-k)} \\
&\quad = (\alpha\beta)^{|n|} \pi^{1/2} \frac{(2|n|)!}{(|n|)!} \sum_{m=0}^{\infty} \frac{(-1)^m \rho^{2m+2|n|} 2^{-(m+|n|)}}{\Gamma(m+|n|+\frac{3}{2})\Gamma(2|n|+m+1)} \mathrm{C}_m^{(|n|+\frac{1}{2})}\left(\frac{\alpha^2+\beta^2}{\alpha^2-\beta^2}\right).
\end{aligned}
\tag{A7}
$$

Combining these results, we find

$$
\begin{aligned}
&\frac{\mu}{(2\pi)^{3/2} K_2(\mu)} \int_{-\infty}^{\infty} \mathrm{d}t \exp(-\mathrm{i}\omega t) \int_{-\infty}^{t} \mathrm{d}t' E_{1i}(\kappa_x', t') \int \frac{\mathrm{d}^3\rho_0}{\gamma^2} \\
&\quad \times \exp\left[-\mu\gamma - \mathrm{i}\frac{c\rho_\perp}{\Omega L}\left\{\kappa_x \sin\left(\phi + \frac{\Omega}{\gamma}t\right) - \kappa_x' \sin\left(\phi + \frac{\Omega}{\gamma}t'\right)\right.\right. \\
&\quad \left.\left. - \kappa_y \cos\left(\phi + \frac{\Omega}{\gamma}t\right) + \kappa_y' \cos\left(\phi + \frac{\Omega}{\gamma}t'\right)\right\}\right] \\
&= \frac{2\mathrm{i}E_{1i}(\kappa', \omega)}{\omega} \sum_{n=-\infty}^{\infty} f_n^{(\frac{3}{2})}(\kappa_\perp, \kappa_\perp') \exp\{\mathrm{i}n(\phi_\kappa - \phi_{\kappa'})\}
\end{aligned}
\tag{A8}
$$

in which $f_n^{(r)}$ is defined by

$$
\begin{aligned}
f_n^{(r)}(\kappa_\perp, \kappa_\perp') &= \left(\frac{\pi\mu}{2}\right)^{1/2} \frac{\exp(-\mu)}{K_2(\mu)} \frac{(2|n|)!}{(|n|)!} \left(\frac{\nu}{2}\right)^{|n|} \sum_{m=0}^{\infty} \frac{(-1)^m\{\kappa_\perp^2 - (\kappa_\perp')^2\}^m}{\Gamma(2|n|+m+1)} \left(\frac{r_c^2}{2L^2}\right)^m \\
&\quad \times \mathrm{C}_m^{(|n|+\frac{1}{2})}\left\{\frac{\kappa_\perp^2 + (\kappa_\perp')^2}{\kappa_\perp^2 - (\kappa_\perp')^2}\right\} \mathcal{F}_{m+|n|+r}\left(\mu, \frac{n\Omega}{\omega}\right).
\end{aligned}
\tag{A9}
$$

We split the expression for the current density into two parts; we write

$$J_{1i}^{(1)}(\boldsymbol{\kappa},\omega) = \frac{e^2\mu^2}{8\pi^2 m L K_2(\mu)} \exp\left(\mathrm{i}\kappa_y \frac{y}{L}\right) \int_{-\infty}^{\infty} \mathrm{d}\kappa_x'\, h(\kappa_x - \kappa_x') \int_{-\infty}^{\infty} \mathrm{d}t \exp(-\mathrm{i}\omega t)$$
$$\times \int_{-\infty}^{t} \mathrm{d}t' \int \frac{\mathrm{d}^3\rho_0}{\gamma^2} \rho_i(t)\rho_j(t') E_j(\kappa_x', t') \exp(-\mu\gamma)$$
$$\times \exp\left[-\mathrm{i}\frac{c\rho_\perp}{\Omega L}\left\{\kappa_x \sin\left(\phi + \frac{\Omega}{\gamma}t\right) - \kappa_x' \sin\left(\phi + \frac{\Omega}{\gamma}t'\right)\right.\right.$$
$$\left.\left. - \kappa_y \cos\left(\phi + \frac{\Omega}{\gamma}t\right) + \kappa_y' \cos\left(\phi + \frac{\Omega}{\gamma}t'\right)\right\}\right] \tag{A10}$$

$$J_{1i}^{(2)}(\boldsymbol{\kappa},\omega) = \frac{\mathrm{i}e^2 c\mu}{8\pi^2 m\Omega L^2 K_2(\mu)} \exp\left(\mathrm{i}\kappa_y \frac{y}{L}\right) \int_{-\infty}^{\infty} \mathrm{d}\kappa_x'\, h(\kappa_x - \kappa_x')(\kappa_x - \kappa_x')$$
$$\times \int_{-\infty}^{\infty} \mathrm{d}t \exp(-\mathrm{i}\omega t) \int_{-\infty}^{t} \mathrm{d}t' \int \frac{\mathrm{d}^3\rho_0}{\gamma} \rho_i(t) \exp(-\mu\gamma)$$
$$\times \left\{E_{1y}(\kappa_x', t') - \frac{c}{\gamma}\rho_x(t') B_{1z}(\kappa_x', t')\right\}$$
$$\times \exp\left[-\mathrm{i}\frac{c\rho_\perp}{\Omega L}\left\{\kappa_x \sin\left(\phi + \frac{\Omega}{\gamma}t\right) - \kappa_x' \sin\left(\phi + \frac{\Omega}{\gamma}t'\right)\right.\right.$$
$$\left.\left. - \kappa_y \cos\left(\phi + \frac{\Omega}{\gamma}t\right) + \kappa_y' \cos\left(\phi + \frac{\Omega}{\gamma}t'\right)\right\}\right]. \tag{A11}$$

Both expressions can be calculated by considering derivatives of (A8) with respect to $\kappa_x, \kappa_x', \kappa_y, \kappa_y'$. In this way we obtain

$$J_{1i}^{(1)} = \frac{\mathrm{i}e^2}{(2\pi)^{1/2}\omega L} \exp\left(\mathrm{i}\kappa_y \frac{y}{L}\right) \int_{-\infty}^{\infty} \mathrm{d}\kappa' Q_{ij}(\kappa,\kappa') E_{1j}(\kappa_x, \kappa_x') \tag{A12}$$

with

$$Q_{xx} = \frac{L^2}{r_c^2} \frac{\partial}{\partial\kappa_y} \frac{\partial}{\partial\kappa_y'} \sum_{n=-\infty}^{\infty} \exp\{\mathrm{i}n(\phi_\kappa - \phi_{\kappa'})\} f_n^{(\frac{3}{2})} \tag{A13}$$

$$Q_{xy} = -\frac{L^2}{r_c^2} \frac{\partial}{\partial\kappa_y} \frac{\partial}{\partial\kappa_x'} \sum_{n=-\infty}^{\infty} \exp\{\mathrm{i}n(\phi_\kappa - \phi_{\kappa'})\} f_n^{(\frac{3}{2})} \tag{A14}$$

$$Q_{yx} = -\frac{L^2}{r_c^2} \frac{\partial}{\partial\kappa_x} \frac{\partial}{\partial\kappa_y'} \sum_{n=-\infty}^{\infty} \exp\{\mathrm{i}n(\phi_\kappa - \phi_{\kappa'})\} f_n^{(\frac{3}{2})} \tag{A15}$$

$$Q_{yy} = \frac{L^2}{r_c^2} \frac{\partial}{\partial\kappa_x} \frac{\partial}{\partial\kappa_x'} \sum_{n=-\infty}^{\infty} \exp\{\mathrm{i}n(\phi_\kappa - \phi_{\kappa'})\} f_n^{(\frac{3}{2})} \tag{A16}$$

$$Q_{zz} = \sum_{n=-\infty}^{\infty} \exp\{\mathrm{i}n(\phi_\kappa - \phi_{\kappa'})\} f_n^{(5/2)} \tag{A17}$$

$$Q_{xz} \equiv Q_{yz} \equiv Q_{zx} \equiv Q_{zy} \equiv 0. \tag{A18}$$

If $k_y = 0$ this reduces to

$$Q_{ij} = \begin{pmatrix} \frac{L^2}{r_c^2}\sum_{n=-\infty}^{\infty}\frac{n^2}{\kappa_x\kappa_x'}f_n^{(\frac{3}{2})} & \frac{L^2}{r_c^2}\sum_{n=-\infty}^{\infty}-\frac{in}{\kappa_x}\frac{\partial f_n^{(\frac{3}{2})}}{\partial\kappa_x'} & 0 \\ \frac{L^2}{r_c^2}\sum_{n=-\infty}^{\infty}\frac{in}{\kappa_x'}\frac{\partial f_n^{(\frac{3}{2})}}{\partial\kappa_x} & \frac{L^2}{r_c^2}\sum_{n=-\infty}^{\infty}\frac{\partial^2 f_n^{(\frac{3}{2})}}{\partial\kappa_x\partial\kappa_x'} & 0 \\ 0 & 0 & \sum_{n=-\infty}^{\infty} f_n^{(5/2)} \end{pmatrix}. \tag{A19}$$

In order to obtain $J_{1i}^{(2)}$ we use the following results:

$$\begin{aligned}\sum_{n=-\infty}^{\infty}&\exp\{in(\phi_\kappa-\phi_{\kappa'})\}J_n\left(\frac{c\kappa_\perp\rho_\perp}{\Omega L}\right)J_n\left(\frac{c\kappa_\perp'\rho_\perp}{\Omega L}\right)\\ &= J_0\left[\frac{c\rho_\perp}{\Omega L}\{\kappa_\perp^2+(\kappa_\perp')^2-2\kappa_\perp\kappa_\perp'\cos(\phi_\kappa-\phi_{\kappa'})\}^{1/2}\right]\\ &= J_0\left\{\frac{c\rho_\perp}{\Omega L}(\kappa_x-\kappa_x')\right\}\end{aligned} \tag{A20}$$

$$\begin{aligned}\int_{-\infty}^{\infty}&d\rho_\parallel\int_0^\infty d\rho_\perp\,\rho_\perp\exp(-\mu\gamma)J_0\left\{\frac{c\rho_\perp}{\Omega L}(\kappa_x-\kappa_x')\right\}\\ &= 2\int_0^{\pi/2}d\psi\cos(\psi)\int_0^\infty dt\,\sinh^2(t)\exp\{-\mu\cosh(t)\}\\ &= \frac{2\Omega L}{c(\kappa_x-\kappa_x')}\int_0^\infty dt\,\sinh(t)\exp\{-\mu\cosh(t)\}\sin\left\{\frac{c}{\Omega L}(\kappa_x-\kappa_x')\sinh(t)\right\}\\ &= \left(2K_1\left[\mu\left\{1+\frac{c^2}{\Omega^2L^2\mu^2}(\kappa_x-\kappa_x')^2\right\}^{1/2}\right]\right)\Big/\left(\mu\left\{1+\frac{c^2}{\Omega^2L^2\mu^2}(\kappa_x-\kappa_x')^2\right\}^{1/2}\right)\end{aligned} \tag{A21}$$

where J_0 is the Bessel function (Magnus *et al* 1966) of the first kind and of order zero and K_1 is the modified Bessel function (Magnus *et al* 1966) of the first kind and of order one. The relation between the Fourier transforms of the functions h and n_0 is calculated in the following way:

$$\begin{aligned}n_0(\kappa_x) &= \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty}dx\exp\left(-i\kappa_x\frac{x}{L}\right)n_0(x)\\ &= \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty}dx\exp\left(-i\kappa_x\frac{x}{L}\right)\int d^3\rho_0\,f^{(0)}(X,\rho)\\ &= \frac{\mu}{2(2\pi)^{3/2}K_2(\mu)}\int_{-\infty}^{\infty}dx\exp\left(-i\kappa_x\frac{x}{L}\right)\int_{-\infty}^{\infty}d\rho_\parallel\int_0^\infty d\rho_\perp\rho_\perp\exp(-\mu\gamma)\\ &\quad\times\int_0^{2\pi}d\phi\,h\left\{x-\frac{c\rho_\perp\sin(\phi)}{\Omega}\right\}\end{aligned}$$

$$
\begin{aligned}
&= \frac{\mu h(\kappa_x)}{2\pi K_2(\mu)} \int_0^\infty \mathrm{d}\rho\, \rho^2 \exp(-\mu\gamma) \int_0^{\pi/2} \mathrm{d}\psi \cos(\psi) \\
&\quad \times \int_0^{2\pi} \mathrm{d}\phi \exp\left\{-\mathrm{i}\frac{c\kappa_x}{\Omega L}\rho\cos(\psi)\sin(\psi)\right\} \\
&= \frac{\mu h(\kappa_x)\Omega L}{c\kappa_x K_2(\mu)} \int_0^\infty \mathrm{d}t \exp\{-\mu\cosh(t)\} \sin\left\{\frac{\kappa_x c}{\Omega L}\sinh(t)\right\} \sinh(t)\cosh(t) \\
&= \frac{\mu h(\kappa_x)\Omega L}{K_2(\mu) c\kappa_x} - \frac{\partial}{\partial\tilde{\mu}}\left[K_1\left\{\left(\tilde{\mu}^2 + \frac{\kappa_x^2 c^2}{\Omega^2 L^2}\right)^{1/2}\right\} \frac{\kappa_x c}{\tilde{\mu}\Omega L}\left(1 + \frac{\kappa_x^2 c^2}{\tilde{\mu}^2\Omega^2 L^2}\right)^{-1/2}\right]_{\tilde{\mu}=\mu}.
\end{aligned} \tag{A22}
$$

Using the recurrence relation for the modified Bessel function (Magnus *et al* 1966) of the second kind

$$
\frac{\mathrm{d}}{\mathrm{d}\tilde{z}}\{\tilde{z}^{-1}K_1(\tilde{z})\}_{\tilde{z}=z} = -z^{-1}K_2(z) \tag{A23}
$$

we have

$$
n_0(\kappa) = h(\kappa)\frac{K_2\{\mu(1 + r_c^2\kappa^2/\mu L^2)^{1/2}\}}{K_2(\mu)(1 + r_c^2\kappa^2/\mu L^2)}\,. \tag{A24}
$$

With the help of these results we find after some algebra

$$
\begin{aligned}
J_x^{(2)}(\kappa,\omega) &= \frac{e^2\Omega}{(2\pi)^{1/2} m\omega^2 L}\exp\left(\mathrm{i}\kappa_y\frac{y}{L}\right)\int_{-\infty}^{\infty}\mathrm{d}\kappa_x'(\kappa_x-\kappa_x')h(\kappa_x-\kappa_x') \\
&\quad \times \sum_{n=-\infty}^{\infty}\exp\{\mathrm{i}n(\phi_\kappa-\phi_{\kappa'})\}\Bigg[E_{1y}(\kappa_x',\omega)\Bigg\{-\frac{n^2}{\kappa_\perp}\cos(\phi_\kappa)\sin^2(\phi_{\kappa'})f_n^{(\frac{3}{2})}(\kappa_\perp,\kappa_\perp') \\
&\quad + \mathrm{i}n\frac{\kappa_\perp'}{\kappa_\perp}\cos(\phi_\kappa)\sin(\phi_{\kappa'})\cos(\phi_{\kappa'})\left(\frac{\partial f_n^{(\frac{3}{2})}(\kappa_\perp,\tilde{\kappa}_\perp')}{\partial\tilde{\kappa}_\perp'}\right)_{\tilde{\kappa}_\perp'=\kappa_\perp'} \\
&\quad + \mathrm{i}n\sin^2(\phi_{\kappa'})\sin(\phi_\kappa)\left(\frac{\partial f_n^{(\frac{3}{2})}(\tilde{\kappa}_\perp,\kappa_\perp')}{\partial\tilde{\kappa}_\perp}\right)_{\tilde{\kappa}_\perp=\kappa_\perp} \\
&\quad + \kappa_\perp'\cos(\phi_{\kappa'})\sin(\phi_{\kappa'})\sin(\phi_\kappa)\left(\frac{\partial^2 f_n(\frac{3}{2})(\tilde{\kappa}_\perp,\tilde{\kappa}_\perp')}{\partial\tilde{\kappa}_\perp\partial\tilde{\kappa}_\perp'}\right)_{\tilde{\kappa}_\perp=\kappa_\perp,\tilde{\kappa}_\perp'=\kappa_\perp'}\Bigg\} \\
&\quad - E_{1x}(\kappa_x',\omega)\Bigg\{\frac{n^2}{\kappa_\perp}\cos(\phi_\kappa)\sin(\phi_{\kappa'})\cos(\phi_{\kappa'})f_n^{(\frac{3}{2})}(\kappa_\perp,\kappa_\perp') \\
&\quad + \mathrm{i}n\cos(\phi_\kappa)\sin(\phi_\kappa)\sin(\phi_{\kappa'})\left(\frac{\partial f_n^{(\frac{3}{2})}(\kappa_\perp,\tilde{\kappa}_\perp')}{\partial\tilde{\kappa}_\perp'}\right)_{\tilde{\kappa}_\perp'=\kappa_\perp'} \\
&\quad - \mathrm{i}n\frac{\kappa_\perp}{\kappa_\perp'}\sin(\phi_\kappa)\cos(\phi_{\kappa'})\sin(\phi_{\kappa'})\left(\frac{\partial f_n^{(\frac{3}{2})}(\tilde{\kappa}_\perp,\kappa_\perp')}{\partial\tilde{\kappa}_\perp}\right)_{\tilde{\kappa}_\perp=\kappa_\perp} \\
&\quad + \kappa_\perp\sin^2(\phi_\kappa)\sin(\phi_{\kappa'})\left(\frac{\partial^2 f_n^{(\frac{3}{2})}(\tilde{\kappa}_\perp,\tilde{\kappa}_\perp')}{\partial\tilde{\kappa}_\perp\partial\tilde{\kappa}_\perp'}\right)_{\tilde{\kappa}_\perp=\kappa_\perp,\tilde{\kappa}_\perp'=\kappa_\perp'}\Bigg\}\Bigg]
\end{aligned} \tag{A25}
$$

$$
\begin{aligned}
J_y^{(2)}(\kappa,\omega) = {} & \frac{\mathrm{i}e^2}{(2\pi)^{1/2} m\omega L} \exp\left(\mathrm{i}\kappa_y \frac{y}{L}\right) \\
& \times \int_{-\infty}^{\infty} \mathrm{d}\kappa'_x \left[\!\!\left[\frac{r_\mathrm{c}^2}{L^2}(\kappa_x - \kappa'_x)^2 n_0(\kappa_x - \kappa'_x) E_{1y}(\kappa'_x,\omega) \right.\right. \\
& + \frac{\Omega}{\omega} h(\kappa_x - \kappa'_x)(\kappa_x - \kappa'_x) \sum_{n=-\infty}^{\infty} \exp\{\mathrm{i}n(\phi_\kappa - \phi_{\kappa'})\} \\
& \times \left[-\mathrm{i}E_{1x}(\kappa'_x,\omega) \left\{ -\frac{n^2}{\kappa_\perp} \sin(\phi_\kappa)\sin(\phi_{\kappa'})\cos(\phi_{\kappa'}) f_n^{(\frac{3}{2})}(\kappa_\perp,\kappa'_\perp) \right.\right. \\
& - \mathrm{i}n \frac{\kappa'_\perp}{\kappa_\perp} \sin(\phi_\kappa)\sin^2(\phi_{\kappa'}) \left(\frac{\partial f_n^{(\frac{3}{2})}(\kappa_\perp,\tilde{\kappa}'_\perp)}{\partial \tilde{\kappa}'_\perp} \right)_{\tilde{\kappa}'_\perp = \kappa'_\perp} \\
& - \mathrm{i}n \cos(\phi_\kappa)\cos(\phi_{\kappa'})\sin(\phi_{\kappa'}) \left(\frac{\partial f_n^{(\frac{3}{2})}(\tilde{\kappa}_\perp,\kappa'_\perp)}{\partial \tilde{\kappa}_\perp} \right)_{\tilde{\kappa}_\perp = \kappa_\perp} \\
& \left. + \kappa'_\perp \cos(\phi_\kappa)\sin^2(\phi_{\kappa'}) \left(\frac{\partial^2 f_n^{(\frac{3}{2})}(\tilde{\kappa}_\perp,\tilde{\kappa}'_\perp)}{\partial\tilde{\kappa}_\perp \partial\tilde{\kappa}'_\perp} \right)_{\tilde{\kappa}_\perp = \kappa_\perp, \tilde{\kappa}'_\perp = \kappa'_\perp} \right\} \\
& + \mathrm{i}E_{1y}(\kappa'_x,\omega) \left\{ \frac{n^2}{\kappa_\perp} \sin(\phi_\kappa)\sin^2(\phi_{\kappa'}) f_n^{(\frac{3}{2})}(\kappa_\perp,\kappa'_\perp) \right. \\
& - \mathrm{i}n \frac{\kappa'_\perp}{\kappa_\perp} \sin(\phi_\kappa)\cos(\phi_\kappa)\sin(\phi_{\kappa'}) \left(\frac{\partial f_n^{(\frac{3}{2})}(\kappa_\perp,\tilde{\kappa}'_\perp)}{\partial \tilde{\kappa}'_\perp} \right)_{\tilde{\kappa}'_\perp = \kappa'_\perp} \\
& + \mathrm{i}n \cos(\phi_\kappa)\sin^2(\phi_{\kappa'}) \left(\frac{\partial f_n^{(\frac{3}{2})}(\tilde{\kappa}_\perp,\kappa'_\perp)}{\tilde{\kappa}_\perp} \right)_{\tilde{\kappa}_\perp = \kappa_\perp} \\
& \left.\left.\left.\left. + \kappa'_\perp \cos(\phi_\kappa)\cos(\phi_{\kappa'})\sin(\phi_{\kappa'}) \left(\frac{\partial^2 f_n^{(\frac{3}{2})}(\tilde{\kappa}_\perp,\tilde{\kappa}'_\perp)}{\partial\tilde{\kappa}_\perp \partial\tilde{\kappa}'_\perp} \right)_{\tilde{\kappa}_\perp = \kappa_\perp, \tilde{\kappa}'_\perp = \kappa'_\perp} \right\} \right] \right]\!\!\right] .
\end{aligned}
$$

In the case of $k_y = 0$, $J_x^{(2)} \equiv 0$ and

$$
J_y^{(2)}(\kappa,\omega) = \frac{\mathrm{i}e^2 r_\mathrm{c}^2}{(2\pi)^{1/2} m\omega L^3} \int_{-\infty}^{\infty} \mathrm{d}\kappa'_x (\kappa_x - \kappa'_x)^2 n_0(\kappa_x - \kappa'_x) E_{1y}(\kappa'_x,\omega) .
$$

Appendix B. Some properties of the relativistic dispersion function $\mathcal{F}_q$

The relativistic dispersion function $\mathcal{F}_q$ defined by (see Bornatici and Ruffina 1988)

$$
\mathcal{F}_q(\mu,\alpha) = \left(\frac{\mu}{2}\right)^{q-1} \frac{1}{\Gamma(q)} \int_1^{\infty} \mathrm{d}\gamma \frac{(\gamma^2-1)^{q-1}}{\gamma-\alpha} \exp\{-\mu(\gamma-1)\} \qquad \alpha \in \mathcal{C} \quad \mu > 0 . \tag{B1}
$$

has proven to be very useful for the mathematical description of relativistic plasmas. Therefore some properties of these functions are given here. Other aspects of the relativistic dispersion function may be found in the literature (Trubnikov and Bazhanova 1959, Shkarofsky 1966, Jacquinot and Leloup 1971, Shkarofsky 1986, Robinson 1987, Sazhin and Temme 1990).

B1. Analyticity

The function $\mathcal{F}_q$ is analytic in the complex α-plane except for the branch-cut $\operatorname{Im}\alpha = 0$, $\operatorname{Re}\alpha \geqslant 1$.

B2. Recurrence relation

The functions $\mathcal{F}_q$ satisfy the following recurrence relation:

$$(q-1)\mathcal{F}_q(\mu,\alpha) = \exp(\mu)\left(\frac{\mu}{2\pi}\right)^{1/2}\{K_{q-\frac{1}{2}}(\mu) + \alpha K_{q-\frac{3}{2}}(\mu)\} + (\alpha^2-1)\frac{\mu}{2}\mathcal{F}_{q-1}(\mu,\alpha) \tag{B2}$$

where K_ν is a modified Bessel function (Magnus *et al* 1966) of the second kind and of order ν.

Proof.

$$\begin{aligned}
\mathcal{F}_q(\mu,\alpha) &= \left(\frac{\mu}{2}\right)^{q-1}\frac{1}{\Gamma(q)}\int_1^\infty \mathrm{d}\gamma\,\frac{(\gamma^2-1)^{q-2}}{\gamma-\alpha}\exp\{-\mu(\gamma-1)\}\{(\gamma^2-\alpha^2)+(\alpha^2-1)\} \\
&= \left(\frac{\mu}{2}\right)^{q-1}\frac{1}{\Gamma(q)}\left\{\int_1^\infty \mathrm{d}\gamma(\gamma^2-1)^{q-2}\gamma\exp\{-\mu(\gamma-1)\}\right. \\
&\quad \left. + (\alpha^2-1)\int_1^\infty \mathrm{d}\gamma\,\frac{(\gamma^2-1)^{q-2}}{\gamma-\alpha}\right\} \\
&= \left(\frac{\mu}{2}\right)^{q-1}\frac{\exp(\mu)}{\Gamma(q)}\left[-\frac{\mathrm{d}}{\mathrm{d}\tilde{\mu}}\left\{\left(\frac{2}{\tilde{\mu}}\right)^{q-3/2}\frac{\Gamma(q-1)}{\pi^{1/2}}K_{q-\frac{3}{2}}(\tilde{\mu})\right\}_{\tilde{\mu}=\mu}\right. \\
&\quad + \alpha\left(\frac{2}{\mu}\right)^{q-3/2}\frac{\Gamma(q-1)}{\pi^{1/2}}K_{q-\frac{3}{2}}(\mu) \\
&\quad \left. + (\alpha^2-1)\Gamma(q-1)\left(\frac{2}{\mu}\right)^{q-2}\exp(-\mu)\mathcal{F}_q(\mu,\alpha)\right] \\
&= \left(\frac{\mu}{2}\right)^{q-1}\frac{\exp(\mu)}{\Gamma(q)}\left\{\frac{\Gamma(q-1)}{\pi^{1/2}}\left(\frac{2}{\mu}\right)^{q-3/2}K_{q-\frac{1}{2}}(\mu)\right. \\
&\quad + \alpha\frac{\Gamma(q-1)}{\pi^{1/2}}\left(\frac{2}{\mu}\right)^{q-3/2}K_{q-\frac{3}{2}}(\mu) \\
&\quad \left. + (\alpha^2-1)\Gamma(q-1)\left(\frac{2}{\mu}\right)^{q-2}\exp(-\mu)\mathcal{F}_{q-1}(\mu,\alpha)\right\}
\end{aligned} \tag{B3}$$

which proves (B2).

B3. Derivative of $\mathcal{F}_q$

The derivative of $\mathcal{F}_q$ with respect to α can be expressed in $\mathcal{F}_q$ and $\mathcal{F}_{q-1}$ as follows:

$$\frac{\partial}{\partial\tilde{\alpha}}\{\mathcal{F}_q(\mu,\tilde{\alpha})\}_{\tilde{\alpha}=\alpha} = -\mu\mathcal{F}_q(\mu,\alpha) + \alpha\mu\mathcal{F}_{q-1}(\mu,\alpha) + \exp(\mu)\left(\frac{2\mu}{\pi}\right)^{1/2} K_{q-\frac{3}{2}}(\mu) . \tag{B4}$$

Proof.

$$\begin{aligned}
\frac{\partial}{\partial\tilde{\alpha}}\{\mathcal{F}_q(\mu,\tilde{\alpha})\}_{\tilde{\alpha}=\alpha} &= \left(\frac{\mu}{2}\right)^{q-1} \frac{1}{\Gamma(q)} \int_1^\infty \mathrm{d}\gamma \frac{(\gamma^2-1)^{q-1}}{(\gamma-\alpha)^2} \exp\{-\mu(\gamma-1)\} \\
&= \left(\frac{\mu}{2}\right)^{q-1} \frac{1}{\Gamma(q)} \left\{ -\frac{(\gamma^2-1)^{q-1}}{\gamma-\alpha} \exp\{-\mu(\gamma-1)\} \,|_1^\infty \right. \\
&\quad \left. + 2\int_1^\infty \mathrm{d}\gamma \frac{(q-1)\gamma(\gamma^2-1)^{q-2}}{\gamma-\alpha} \exp\{-\mu(\gamma-1)\} \right\} \\
&\quad - \mu \int_1^\infty \mathrm{d}\gamma \frac{(\gamma^2-1)^{q-1}}{\gamma-\alpha} \exp(-\mu\gamma) \\
&= -\mu\mathcal{F}_q(\mu,\alpha) + \alpha\mu\mathcal{F}_{q-1}(\mu,\alpha) + \exp(\mu)\left(\frac{2\mu}{\pi}\right)^{1/2} K_{q-\frac{3}{2}}(\mu) .
\end{aligned} \tag{B5}$$

B4. Asymptotics

Introducing (see Bornatici and Ruffina (1988))

$$\begin{aligned}
F_q(\mu,\alpha) &= \frac{\mu^{q-1}}{\Gamma(q)} \int_0^\infty \mathrm{d}x \frac{x^{q-1}}{x+1-\alpha} \exp(-\mu x) \\
&= \frac{1}{\Gamma(q)} \int_0^\infty \mathrm{d}x \frac{x^{q-1}\exp(-x)}{x+\mu(1-\alpha)}
\end{aligned} \tag{B6}$$

the following asymptotic expansion of $\mathcal{F}$ can be derived:

$$\begin{aligned}
\mathcal{F}_q(\mu,\alpha) &\sim \sum_{n=0}^{\infty} \frac{\Gamma(q+n)}{n!\Gamma(q-n)} \frac{1}{(2\mu)^n} F_{q+n}(\mu,\alpha) \\
&= \sum_{n=0}^{N} \frac{\Gamma(q+n)}{n!\Gamma(q-n)} \frac{1}{(2\mu)^n} F_{q+n}(\mu,\alpha) + \mathrm{O}\left(\mu^{-(N+1)}\right) .
\end{aligned} \tag{B7}$$

Proof. We have

$$\begin{aligned}
\mathcal{F}_q(\mu,\alpha) &= \left(\frac{\mu}{2}\right)^{q-1} \frac{\exp(\mu)}{\Gamma(q)} \int_1^\infty \mathrm{d}\gamma \frac{(\gamma^2-1)^{q-1}}{\gamma-\alpha} \exp(-\mu\gamma) \\
&= \left(\frac{\mu}{2}\right)^{q-1} \frac{1}{\Gamma(q)} \int_0^\infty \mathrm{d}x \frac{x^{q-1}(x+2)^{q-1}}{x+1-\alpha} \exp(-\mu x) \\
&\sim \frac{\mu^{q-1}2^q}{\Gamma(q)} \int_0^\infty \mathrm{d}x \frac{x^{q-1}(x+1)^{q-1}}{2x+1-\alpha} \exp(-2\mu x) \\
&= \frac{\mu^{q-1}2^{q-1}}{\Gamma(q)} \sum_{n=0}^{\infty} \binom{q}{n} \int_0^\infty \mathrm{d}x \frac{x^{q+n-1}}{x+1-\frac{\alpha}{2}} \exp(-2\mu x) \\
&= \frac{\mu^{q-1}}{\Gamma(q)} \sum_{n=0}^{\infty} 2^{-n} \binom{q}{n} \int_0^\infty \mathrm{d}x \frac{x^{q+n-1}}{x+1-\alpha} \exp(-\mu x) .
\end{aligned} \tag{B8}$$

An alternative integral representation for the function $F_q(\mu, \alpha)$ may be obtained by replacing the factor $\{x + \mu(1-\alpha)\}^{-1}$ in (B6) by $-\mathrm{i}\int_0^\infty \mathrm{d}t \exp[\mathrm{i}\{x + \mu(1-\alpha)\}t]$ and exchanging the integration order. Thus it is found that

$$F_q(\mu, \alpha) = -\mathrm{i}\int_0^\infty \mathrm{d}t \frac{\exp\{\mathrm{i}\mu(1-\alpha)t\}}{(1-\mathrm{i}t)^q} \tag{B9}$$

which is known as the Dnestrovskii function (see Dnestrovskii *et al* 1964).

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